

Unit 5: Circular Functions ~ Learning Guide

Name: _____

Instructions:

Using a pencil, complete the following questions as you work through the related lessons.
Show ALL work as it is explained in the lessons. Do your best and ask your instructor if you don't understand any questions!

Lesson One

Radian Measure and Angles in Standard Position:

1. Define the following terms.

a) Standard Position Angle:

An angle drawn on the x-y plane
with vertex located at the origin and rotating the desired amount and
direction.

b) Initial Arm:

+ve x-axis (0° angle)

c) Terminal Arm:

The line drawn for an angle
in standard position.

d) Co-Terminal Angle

An angle with a different numerical
value but same standard position.

2. Convert each of the following into radians exactly (in terms of π).

a) 30°

$$\begin{aligned}\pi &= 180^\circ \\ 1^\circ &= \frac{\pi}{180} \\ 30^\circ &= \frac{\pi}{180} \times 30 \\ &= \frac{\pi}{6}\end{aligned}$$

b) 240°

$$\begin{aligned}180^\circ &= \pi \\ 1^\circ &= \frac{\pi}{180} \\ 240^\circ &= \frac{\pi}{180} \times 240 \\ &= \frac{4\pi}{3}\end{aligned}$$

c) 110°

$$\begin{aligned}180^\circ &= \pi \\ 1^\circ &= \frac{\pi}{180} \\ 110^\circ &= \frac{\pi}{180} \times 110 \\ &= \frac{11\pi}{18}\end{aligned}$$

d) -10°

$$\begin{aligned}180^\circ &= \pi \\ 1^\circ &= \frac{\pi}{180} \\ -10^\circ &= \frac{\pi}{180} \times -10 \\ &= \frac{-\pi}{18}\end{aligned}$$

3. Convert the following to degrees exactly.

a) $\frac{\pi}{3}$

$$\pi = 180^\circ$$

$$\frac{\pi}{3} = \frac{180}{3}$$

$$= 60^\circ$$

b) $\frac{5\pi}{6}$

$$\pi = 180^\circ$$

$$\frac{5\pi}{6} = 180 \times \frac{5}{6}$$

$$= 150^\circ$$

c) $\frac{\pi}{10}$

$$\pi = 180^\circ$$

$$\frac{\pi}{10} = \frac{180}{10}$$

$$= 18^\circ$$

d) $-\frac{5\pi}{9}$

$$\pi = 180^\circ$$

$$-\frac{5\pi}{9} = 180 \times \left(-\frac{5}{9}\right)$$

$$= -100^\circ$$

4. Determine one positive and one negative co-terminal angle for the following angles.

a) 140°

$$\rightarrow 140 - 360$$

$$= -220^\circ$$

$$\rightarrow 140 + 360$$

$$= 500^\circ$$

b) $\frac{9\pi}{7}$

$$\rightarrow \frac{9\pi}{7} - 2\pi$$

$$= -\frac{5\pi}{7}$$

$$\rightarrow \frac{9\pi}{7} + 2\pi$$

$$= \frac{23\pi}{7}$$

c) -78°

$$\rightarrow -78 - 360$$

$$= -438^\circ$$

$$\rightarrow -78 + 360$$

$$= 282^\circ$$

d) $-\frac{6\pi}{5}$

$$-\frac{6\pi}{5} - 2\pi = -\frac{16\pi}{5}$$

$$-\frac{6\pi}{5} + 2\pi = \frac{4\pi}{5}$$

5. Determine the length of an arc in a circle of radius 4cm if it subtends an angle of 2rads.

$$r = 4 ; \theta = 2 \text{ rad}$$

$$\text{Arc length} = r\theta = 4 \times 2 = 8 \text{ cm}$$

6. Determine the radius length in a circle if an arc of length 5cm subtends an angle of 20° .

$$\text{Arc length} = 5 \text{ cm}$$

$$\theta = 20^\circ \Rightarrow \frac{\pi}{180} \times 20 = \frac{\pi}{9} = 0.349 \text{ rad}$$

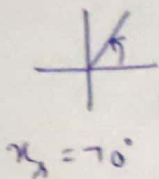
$$\text{Arc length} = r\theta \Rightarrow r = \frac{5}{0.349} = 14.32 \text{ cm}$$

Lesson TwoSine, Cosine and Tangent:

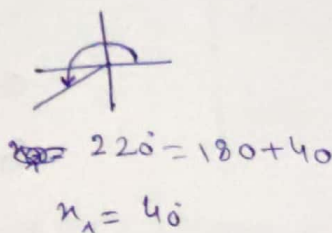
1. Define Reference Angle: The smallest +ve angle between the terminal arm and x-axis.

2. Draw each angle in standard position and determine its reference angle.

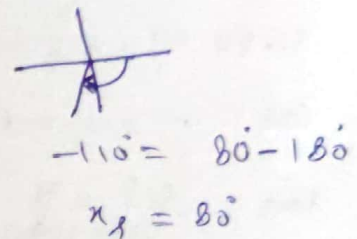
a) $x = 70^\circ$



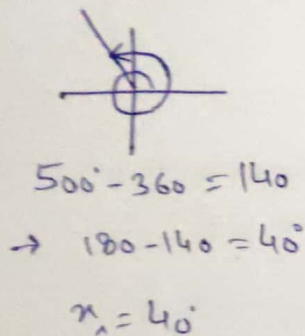
b) $x = 220^\circ$



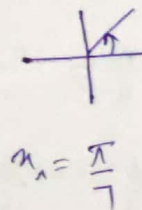
c) $x = -100^\circ$



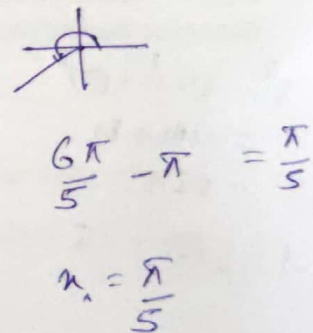
d) $x = 500^\circ$



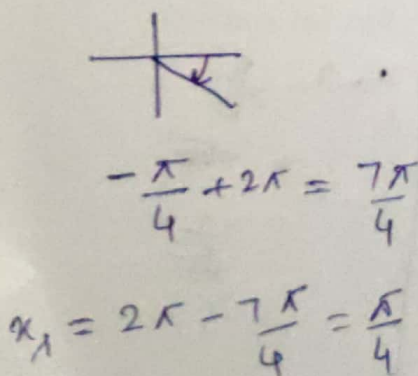
e) $\theta = \frac{\pi}{7}$



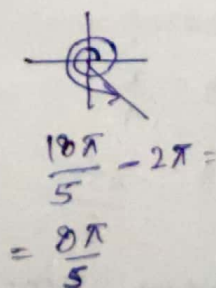
f) $\theta = \frac{6\pi}{5}$



g) $\theta = -\frac{\pi}{4}$



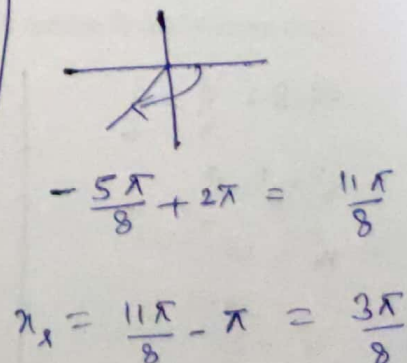
h) $\theta = \frac{18\pi}{5}$



$x_r = 2\pi - \frac{8\pi}{5} = \frac{2\pi}{5}$

Page 3 of 27

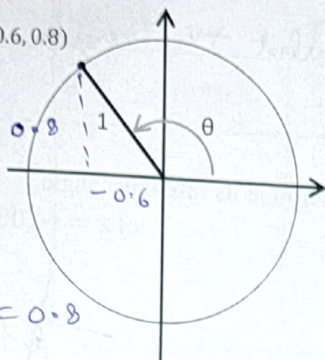
i) $\theta = -\frac{5\pi}{8}$



3. For each diagram determine $\sin\theta$, $\cos\theta$ and $\tan\theta$.

a)

$P(-0.6, 0.8)$

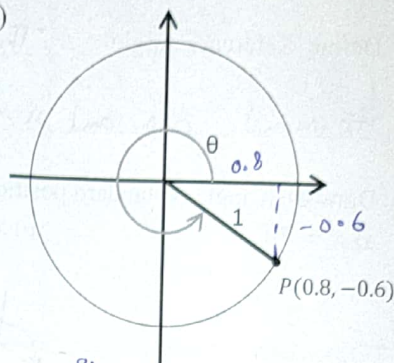


$$\sin\theta = \frac{0.8}{1} = 0.8$$

$$\cos\theta = \frac{-0.6}{1} = -0.6$$

$$\tan\theta = \frac{0.8}{-0.6} = -\frac{4}{3}$$

b)



$$\sin\theta = \frac{-0.6}{1} = -0.6$$

$$\cos\theta = \frac{0.8}{1} = 0.8$$

$$\tan\theta = \frac{-0.6}{0.8} = -\frac{3}{4}$$

4. Point $A(-12, -9)$ is in Quadrant III and lies on the terminal arm of the standard position angle θ . Point P is the point of intersection of the terminal arm of θ and the unit circle. Determine the coordinates of P .

$$r^2 = (-12)^2 + (-9)^2$$

$$= 144 + 81$$

$$= 225$$

$$r = \sqrt{225} = 15$$

$$\cos\theta = \frac{x}{r} = \frac{-12}{15} = -\frac{4}{5}$$

$$\sin\theta = \frac{y}{r} = \frac{-9}{15} = -\frac{3}{5}$$

$$P = \left(-\frac{4}{5}, -\frac{3}{5}\right)$$

5. Given that the angle θ is in standard position with its terminal arm in quadrant two, find the exact value of $\cos\theta$ and $\tan\theta$ if $\sin\theta = \frac{1}{6}$.

$$\sin\theta = \frac{y}{r} = \frac{1}{6}$$

$$x^2 + y^2 = r^2$$

$$\Rightarrow x^2 = (6)^2 - (1)^2$$

$$= 36 - 1$$

$$x^2 = 35$$

$$x = \sqrt{35}$$

$$\cos\theta = \frac{x}{r}$$

$$\cos\theta = -\frac{\sqrt{35}}{6}$$

Because quadrant two

$$\tan\theta = \frac{y}{x}$$

$$= \frac{1}{\sqrt{35}}$$

$$= -\frac{1}{\sqrt{35}}$$

6. Find the exact value of $\sin \theta$, $\cos \theta$ and $\tan \theta$ if the terminal arm of the standard position angle θ contains the point

a) $(-2, -5)$

$$r^2 = x^2 + y^2 = (-2)^2 + (-5)^2 = 29$$

$$r = \sqrt{29}$$

$$\sin \theta = \frac{y}{r} = \frac{-5}{\sqrt{29}}$$

$$\cos \theta = \frac{x}{r} = \frac{-2}{\sqrt{29}}$$

$$\tan \theta = \frac{y}{x} = \frac{-5}{-2}$$

b) $(-3, 7)$

$$r^2 = x^2 + y^2 = (-3)^2 + (7)^2 = 58$$

$$r = \sqrt{58}$$

$$\sin \theta = \frac{y}{r} = \frac{7}{\sqrt{58}}$$

$$\cos \theta = \frac{x}{r} = \frac{-3}{\sqrt{58}}$$

$$\tan \theta = \frac{y}{x} = \frac{-7}{3}$$

7. Given that $\cos \theta = \frac{2}{3}$ and that $\sin \theta < 0$, determine the exact value of $\sin \theta$.

$$\sin \theta = -\sqrt{1 - \cos^2 \theta}$$

$$= -\sqrt{1 - \left(\frac{2}{3}\right)^2}$$

$$= -\sqrt{1 - \frac{4}{9}} = -\sqrt{\frac{5}{9}}$$

8. Given that $\sin \theta = -\frac{1}{4}$ and that $\cos \theta < 0$, determine the exact value of $\tan \theta$

$$\cos \theta = -\sqrt{1 - \sin^2 \theta}$$

$$= -\sqrt{1 - \left(-\frac{1}{4}\right)^2}$$

$$= -\sqrt{1 - \frac{1}{16}} = -\sqrt{\frac{15}{16}}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{-\frac{1}{4}}{-\sqrt{\frac{15}{16}}}$$

9. Given that $\sin \theta = a$, with $0 < a$ and that $\tan \theta < 0$, determine $\cos \theta$.

$$\cos \theta = -\sqrt{1 - \sin^2 \theta}$$

$$= -\sqrt{1 - a^2}$$

10. Given that $\tan \theta = -\frac{3}{2}$ with $\frac{\pi}{2} < \theta < \pi$, determine the exact value of $\cos \theta$.

$\theta = \text{Quadrant II}$

$$\cos \theta < 0$$

$$\tan \theta = \frac{y}{x} = -\frac{3}{2}$$

$$x^2 + y^2 = r^2 \Rightarrow r^2 = (3)^2 + (2)^2 = 13$$

$$r = \sqrt{13}$$

$$\cos \theta = \frac{x}{r}$$

$$\cos \theta = \frac{-2}{\sqrt{13}}$$

11. If $\cos \theta = \frac{3}{4}$ with $0 < \theta < \frac{\pi}{2}$, then determine the exact value of $\sin(\pi + \theta)$

$\theta = \text{I Quadrant}$

$\pi + \theta = \text{III Quadrant} \Rightarrow \sin \theta < 0$

$$\cos \theta = \frac{3}{4} = \frac{x}{r}$$

$$x^2 + y^2 = r^2$$

$$y^2 = r^2 - x^2 = (4)^2 - (3)^2 = 7$$

$$y = \sqrt{7}$$

$$\sin \theta = \frac{y}{r}$$

$$= -\frac{\sqrt{7}}{4}$$

Lesson Three

Sine, Cosine and Tangent of Angles:

1. What is the purpose of the phrase "All students take calculus"?

It is mnemonic for sign of each trigonometric function in each quadrant.

2. Using your calculator, evaluate each of the following trigonometric ratios to the nearest thousandth.

a) $\sin 3.45$

$$-0.304$$

b) $3 \tan 2.40$

$$3 \times (-0.9160)$$

$$= -2.748$$

c) $\frac{2}{\cos 4.67}$

$$\frac{2}{-0.04238}$$

$$= -47.196$$

d) $\csc 2.65$

$$\frac{1}{\sin 2.65}$$

$$= \frac{1}{0.47203} = 2.119$$

3. Determine the exact value for the following questions (do not use a calculator).

a) $\sin \frac{\pi}{3}$

$$\frac{\sqrt{3}}{2}$$

b) $\cos \frac{3\pi}{4}$

$$\sin\left(\frac{\pi}{2} - \frac{3\pi}{4}\right)$$

$$\Rightarrow \sin\left(-\frac{\pi}{4}\right)$$

$$= -\sin \frac{\pi}{4} = -\frac{1}{\sqrt{2}}$$

c) $-\tan \frac{5\pi}{6}$

$$= -\tan\left(\pi - \frac{\pi}{6}\right)$$

$$= -\left(-\tan \frac{\pi}{6}\right)$$

$$\tan \frac{\pi}{6}$$

$$\frac{1}{\sqrt{3}}$$

e) $\cos \frac{3\pi}{2}$

$$\cos\left(\pi + \frac{\pi}{2}\right)$$

$$= -\cos \frac{\pi}{2}$$

$$= 0$$

g) $3 \tan \frac{5\pi}{3}$

$$3 \times \tan\left(\pi + \frac{2\pi}{3}\right)$$

$$= 3 \times \tan\left(\frac{2\pi}{3}\right)$$

$$= 3 \times (-\sqrt{3})$$

$$= -3\sqrt{3}$$

i) $\cot \frac{5\pi}{6}$

$$\cot\left(\pi - \frac{\pi}{6}\right)$$

$$= -\cot \frac{\pi}{6}$$

$$= -\sqrt{3}$$

d) $\sin \frac{7\pi}{4}$

$$\sin\left(2\pi - \frac{\pi}{4}\right)$$

$$= -\sin \frac{\pi}{4}$$

$$= -\frac{1}{\sqrt{2}}$$

f) $\csc \frac{\pi}{4}$

$$\frac{1}{\sin \frac{\pi}{4}} = \sqrt{2}$$

h) $\sin\left(-\frac{3\pi}{4}\right)$

$$= -\sin\left(\frac{3\pi}{4}\right)$$

$$= -\sin\left(\pi - \frac{\pi}{4}\right)$$

$$= -\sin \frac{\pi}{4}$$

$$= -\frac{1}{\sqrt{2}}$$

j) $4 \sec \frac{11\pi}{6}$

$$4 \times \sec\left(2\pi - \frac{\pi}{6}\right)$$

$$4 \times \sec \frac{\pi}{6}$$

$$= 4 \times \frac{2}{\sqrt{3}}$$

$$= \frac{8\sqrt{3}}{3}$$

$$k) 8 \sec\left(-\frac{7\pi}{6}\right)$$

$$8 \sec\left(\frac{7\pi}{6}\right)$$

$$8 \sec\left(\pi + \frac{\pi}{6}\right)$$

$$-8 \sec\left(\frac{\pi}{6}\right)$$

$$= -8 \times \frac{2}{\sqrt{3}}$$

$$= -\frac{16}{\sqrt{3}}$$

$$m) 2 \csc \frac{5\pi}{2}$$

$$2 \csc\left(2\pi + \frac{\pi}{2}\right)$$

$$2 \csc\left(\frac{\pi}{2}\right)$$

$$2$$

$$o) \log_3\left(\tan \frac{\pi}{3}\right)$$

$$\log_3 \sqrt{3}$$

$$\log_3 (3)^{\frac{1}{2}}$$

$$= \frac{1}{2}$$

$$l) -2 \sin \frac{3\pi}{4} + \cos \frac{3\pi}{4}$$

$$-2 \times \frac{1}{\sqrt{2}} + \left(-\frac{1}{\sqrt{2}}\right)$$

$$= -\frac{3\sqrt{2}}{2}$$

$$n) 2 \tan \frac{5\pi}{6} - \csc \frac{10\pi}{3}$$

$$2 \tan\left(\pi - \frac{\pi}{6}\right) - \csc\left(3\pi + \frac{\pi}{3}\right)$$

$$= -2 \tan \frac{\pi}{6} - \left(-\csc\left(\frac{\pi}{3}\right)\right)$$

$$= -\frac{2}{\sqrt{3}} + \frac{2}{\sqrt{3}}$$

$$= 0$$

$$p) \log_3\left(\tan \frac{7\pi}{6}\right)$$

$$= \log_3\left(\tan\left(\pi + \frac{\pi}{6}\right)\right)$$

$$= \log_3\left(\tan \frac{\pi}{6}\right)$$

$$= \log_3\left(\frac{1}{\sqrt{3}}\right)$$

$$= \log_3 1 - \log_3 (3)^{\frac{1}{2}}$$

$$= -\frac{1}{2}$$

$$\begin{aligned}
 \text{q) } \log_2 \left(\sin \frac{3\pi}{4} \right) \\
 &= \log_2 \left(\sin \left(\pi - \frac{\pi}{4} \right) \right) \\
 &= \log_2 \left(\sin \frac{\pi}{4} \right) \\
 &= \log_2 \left(\frac{1}{\sqrt{2}} \right) \\
 &= -\frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{r) } \log_2 \left(\sec \left(-\frac{\pi}{3} \right) \right) \\
 &= \log_2 2 \\
 &= 1
 \end{aligned}$$

4. Determine in which quadrant the standard position angle θ is located when:

a) $0 < \sin \theta$ and $\sec \theta < 0$

Quadrant II

b) $\cot \theta < 0$ and $\csc \theta < 0$

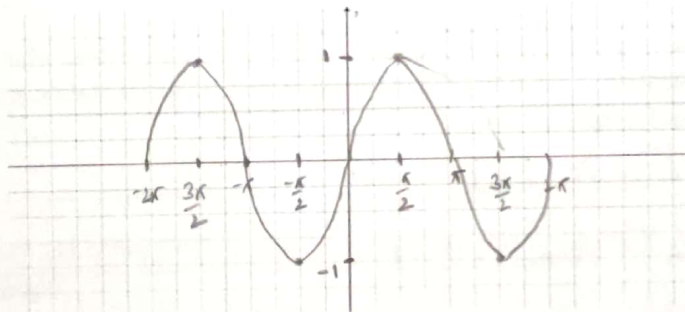
Quadrant IV

Lesson Four

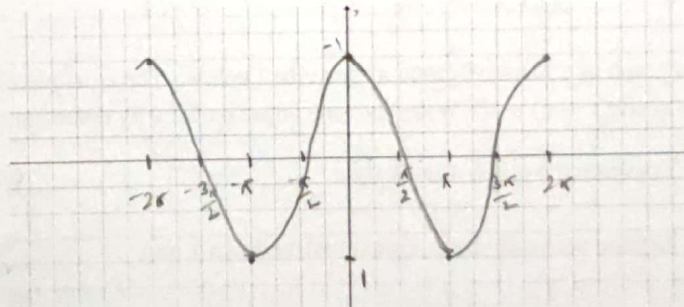
Graphing Sine and Cosine Functions:

1. Without using a calculator or making a table of values, graph the desired function over the interval $-2\pi \leq \theta \leq 2\pi$.

a) $y = \sin \theta$



b) $y = \cos \theta$



2. For the graph of $y = \sin \theta$ graphed on the interval $0 \leq \theta \leq 2\pi$

a) What is the maximum value of y ? For what values of θ does this occur?

$$y = \sin \theta \quad \text{Max Value} = 1; \quad \theta = \frac{\pi}{2}$$

b) What is the minimum value of y ? For what values of θ does this occur?

$$\text{Min} = -1 \quad \theta = \frac{3\pi}{2}$$

c) Determine the θ -intercepts.

$$\theta \text{ intercept} \Rightarrow y = \sin \theta = 0 \Rightarrow \theta = 0, \pi, 2\pi$$

3. For the graph of $y = \cos \theta$ graphed on the interval $0 \leq \theta \leq 2\pi$

a) What is the maximum value of y ? For what values of θ does this occur?

$$y = \cos \theta \quad \text{Max} = 1, \quad \theta = 0, 2\pi$$

b) What is the minimum value of y ? For what values of θ does this occur?

$$\text{Min} = -1, \quad \theta = \pi$$

c) Determine the θ -intercepts.

$$\theta \text{ intercept} \Rightarrow y = \cos \theta = 0 \Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

Lesson Five

Transformations of Trigonometric Functions I:

1. Given the sinusoidal function $y = a \cos(x - c) + d$, discuss what the effect of each of the variables a , c and d have on the graph (Think of "trigonometric graphing" terms rather than "transformations" terms).

a) a : Amplitude

b) c : Phase Shift

c) d : Location of Central axis

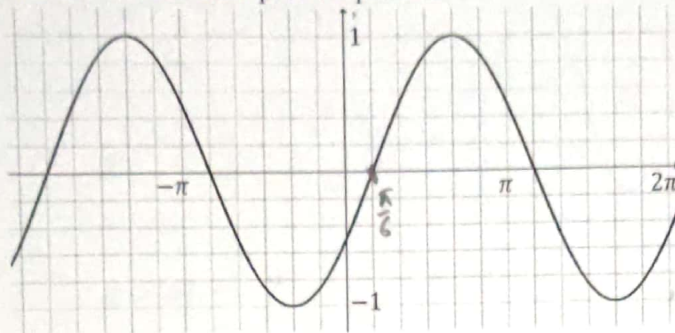
2. The graph of y is transformed as described below. The equation of its image has the form $y = a \sin(x - c) + d$. Write the new equation for y by determining a , c and d .

a) Translate the graph 4 units right $y = \sin(x - 4)$

b) Expand vertically by 3, then translate down 1 unit $y = 3 \sin x - 1$

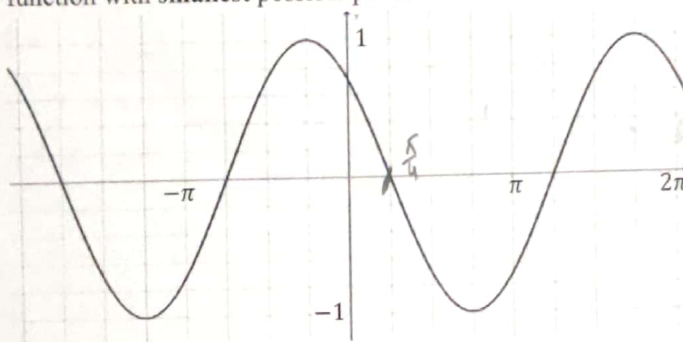
c) Compress vertically by $\frac{1}{2}$, then translate $\frac{\pi}{7}$ units left $y = \frac{1}{2} \sin(x + \frac{\pi}{7})$

3. The graph below can be considered as a sine function. Determine the equation of this function with **smallest** possible phase shift.



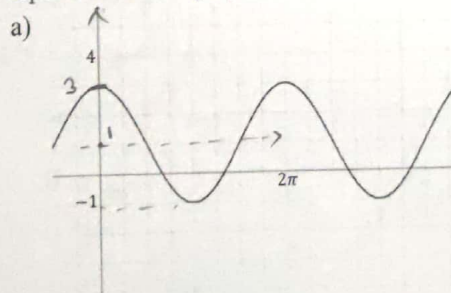
$$y = \sin\left(x - \frac{\pi}{6}\right)$$

4. The graph below can be considered as a cosine function. Determine the equation of this function with **smallest** possible phase shift.

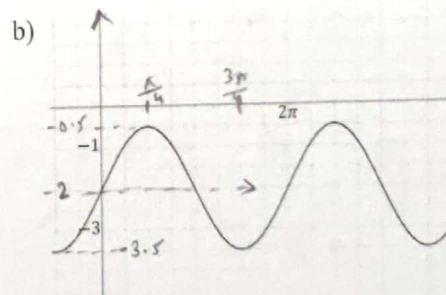


$$y = \cos\left(x + \frac{\pi}{4}\right)$$

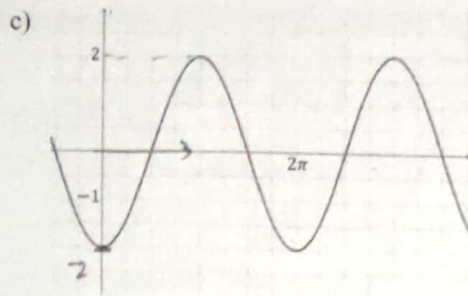
5. Each function below has either the form $y = a \sin x + d$ or $y = a \cos x + d$. Write the equation of each graph.



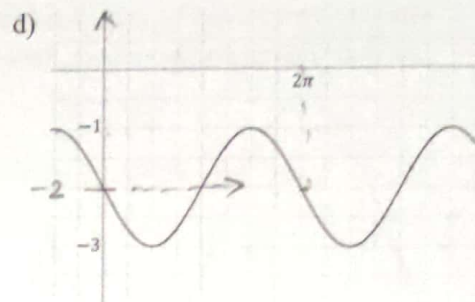
$$y = 2 \cos x + 1$$



$$y = 1.5 \sin x - 2$$



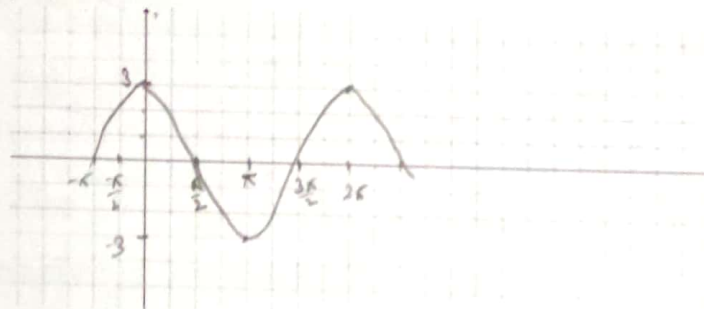
$$y = -2 \cos x$$



$$y = -\sin x - 2$$

6. Graph at least one cycle of each function. Determine its domain, range, phase shift and amplitude.

a) $y = 3 \cos x$



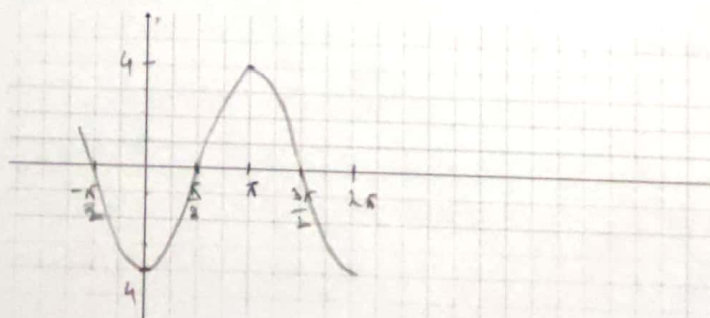
Domain: $x \in \mathbb{R}$

Range: $-3 \leq y \leq 3$

P.S. = 0

amp. = 3

b) $y = -4 \cos x$



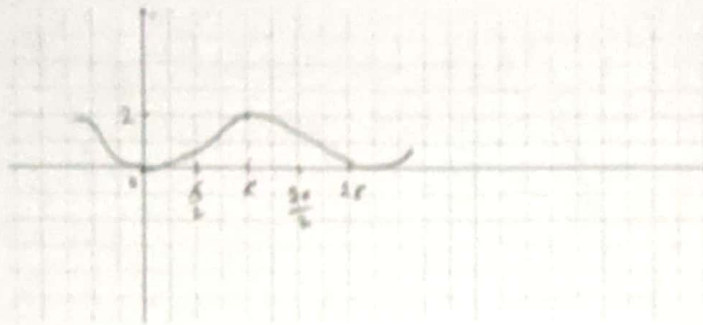
Domain: $x \in \mathbb{R}$

Range: $-4 \leq y \leq 4$

P.S. = 0

amp. = 4

c) $y = -\cos x + 1$



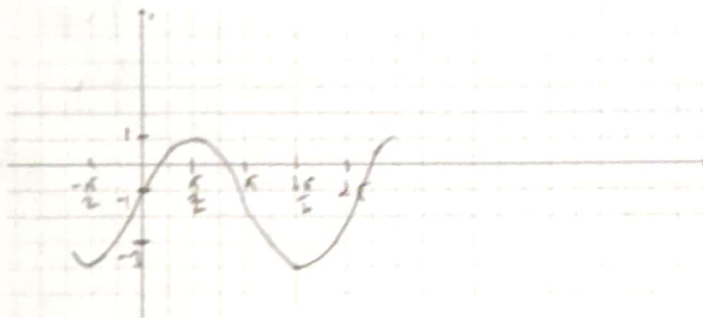
Domain: $x \in \mathbb{R}$

Range: $0 \leq y \leq 2$

P.S.: π

Amplitude: 1

d) $y = 2\sin x - 1$



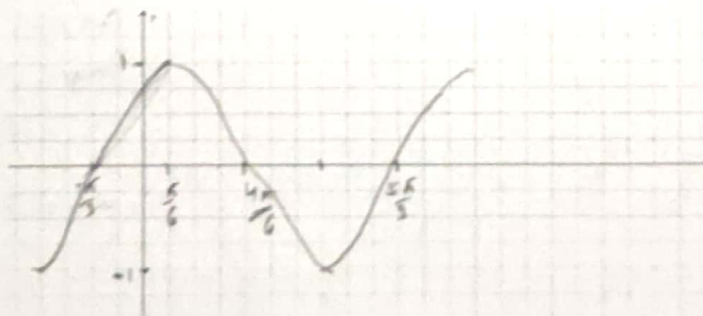
Domain: $x \in \mathbb{R}$

Range: $-3 \leq y \leq 1$

P.S.: π

Amplitude: 2

e) $y = \cos\left(x - \frac{\pi}{6}\right)$



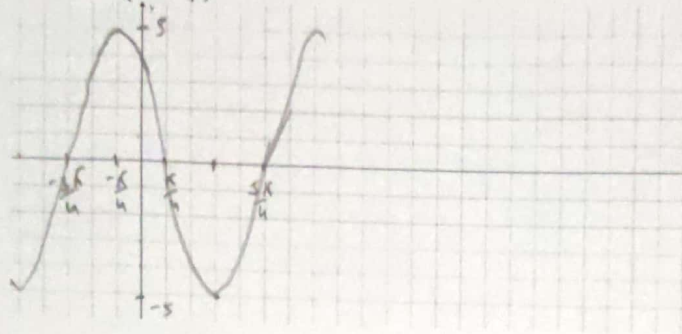
Domain: $x \in \mathbb{R}$

Range: $-1 \leq y \leq 1$

P.S.: $= \frac{\pi}{6}$ right

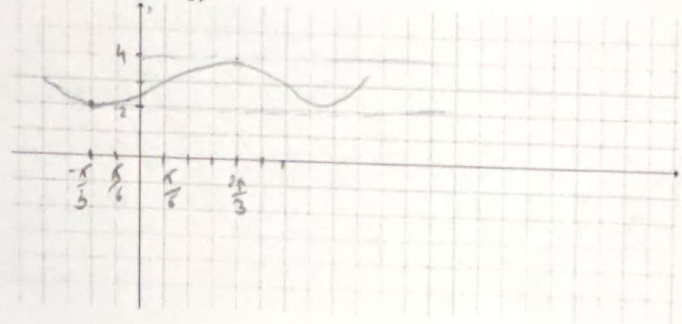
Amplitude: 1

f) $y = 5 \sin\left(x + \frac{3\pi}{4}\right)$



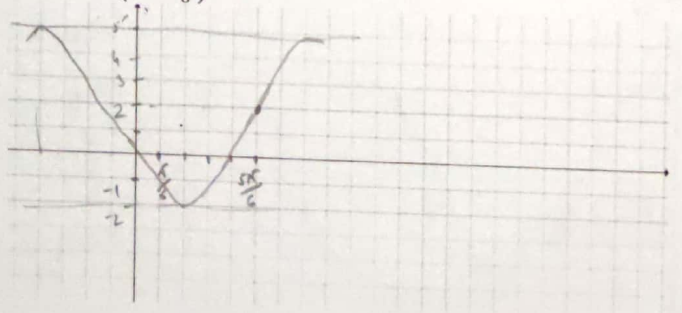
Domain: $x \in \mathbb{R}$
 Range: $-5 \leq y \leq 5$
 P.S. = $\frac{3\pi}{4}$ left
 amp = 5

g) $y = -\cos\left(x + \frac{\pi}{3}\right) + 3$



Domain: $x \in \mathbb{R}$
 Range: $2 \leq y \leq 4$
 P.S. = $\frac{\pi}{3}$ left
 amp = 1

h) $y = 3 \sin\left(x - \frac{5\pi}{6}\right) + 2$



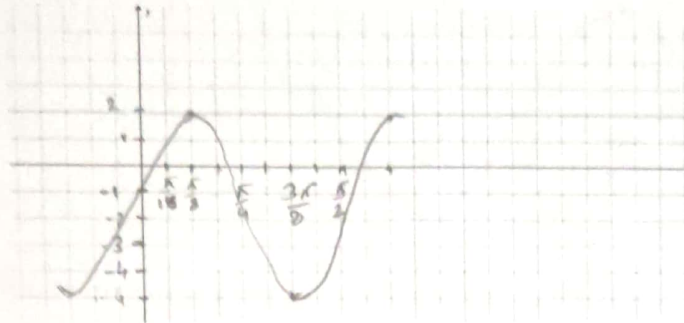
Domain: $x \in \mathbb{R}$
 Range: $-1 \leq y \leq 5$
 P.S. = $\frac{5\pi}{6}$ right
 amp = 3

Lesson Six

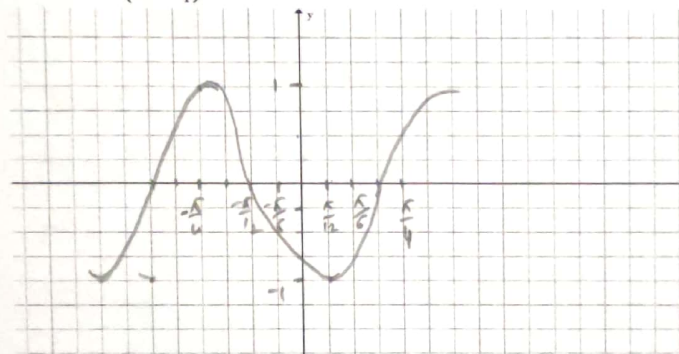
Transformations of Trigonometric Functions II:

1. Graph at least one cycle of each function.

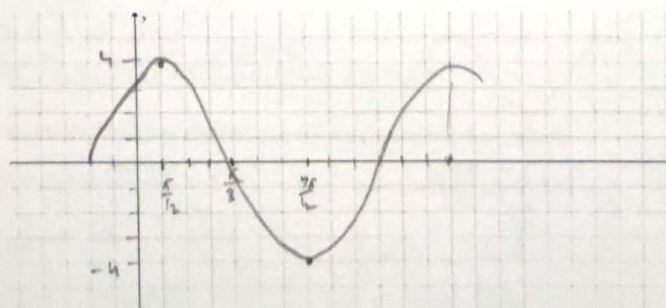
a) $y = 3 \sin 4x - 1$



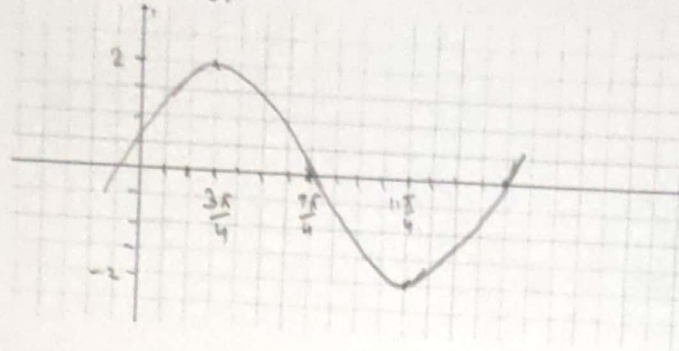
b) $y = \cos 3\left(x + \frac{\pi}{4}\right)$



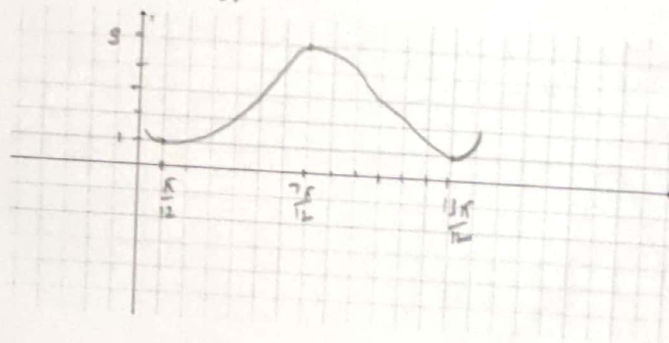
c) $y = -4 \sin 2\left(x - \frac{\pi}{3}\right)$



d) $y = 2 \cos\left(\frac{1}{2}x - \frac{3\pi}{8}\right)$

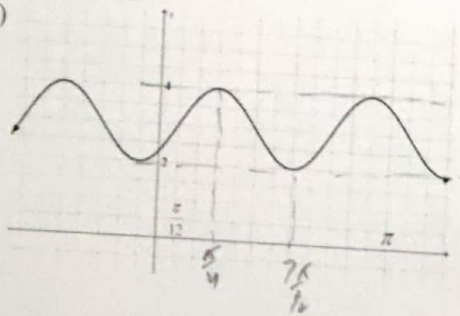


e) $y = -2\sin 2\left(x - \frac{5\pi}{6}\right) + 3$



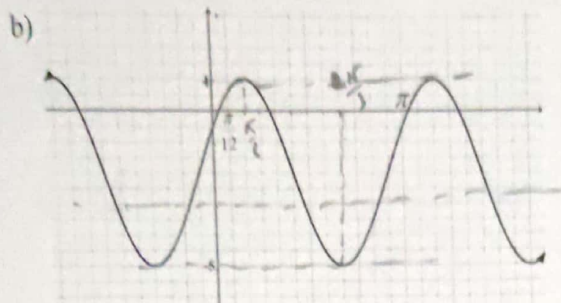
2. Determine both the cosine and sine equation, with smallest possible phase shift, for the following graphs.

a)



$$y = -\cos 3\left(x + \frac{\pi}{12}\right) + 3$$

$$y = \sin 3\left(x - \frac{\pi}{12}\right) + 3$$



$$y = 3 \cos 2 \left(x - \frac{\pi}{6} \right) - 2$$

$$y = 3 \sin 2 \left(x + \frac{\pi}{6} \right) - 2$$

3. Write the equation of a sine curve, with smallest possible phase shift, if it has a maximum at $\left(\frac{\pi}{12}, 13\right)$ and the nearest minimum to the right at $\left(\frac{7\pi}{12}, 3\right)$.

$$y = a \sin(x - c) + d$$

$$\text{Max} - \text{Min} = \text{location of Central axis} = \frac{13 + 3}{2} = 8$$

$$y = a \sin(x - c) + 8$$

$$\text{Max} \Rightarrow 13 = a \sin\left(\frac{\pi}{12} - c\right)$$

$$\text{Min} = 3 = a \sin\left(\frac{7\pi}{12} - c\right)$$

Solving

$$y = 5 \sin\left(2\left(x + \frac{\pi}{6}\right)\right) + 8$$

Lesson Seven

Sinusoidal Functions with Rational Periods:

1. Given each function determine the period and phase shift.

a) $y = 2 \sin \frac{2\pi(x-3)}{5} + 1$

Period = 5

P.S. = 3 right

b) $y = -\cos\left(\frac{2\pi}{12}(x+1)\right)$

Period = 12

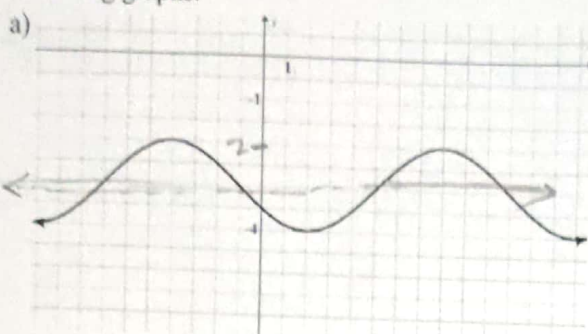
P.S. = 1 left

c) $y = 3 \sin \frac{\pi(x+2)}{7} - 5$

Period = 14

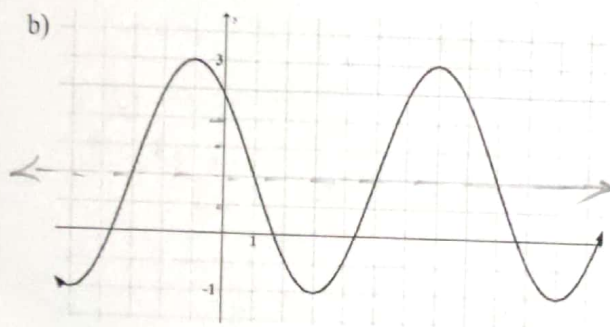
P.S. = 2 left

2. Determine both the cosine and sine equation, with smallest possible phase shift, for the following graphs.



$$y = -2 \cos \left(\frac{2\pi(x-2)}{12} \right) - 3$$

$$y = -2 \sin \left(\frac{2\pi(x+1)}{12} \right) - 3$$

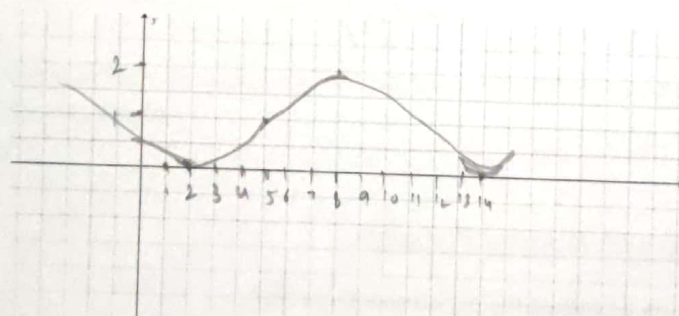


$$y = 2 \cos \left[\frac{2\pi(x+1)}{8} \right] + 1$$

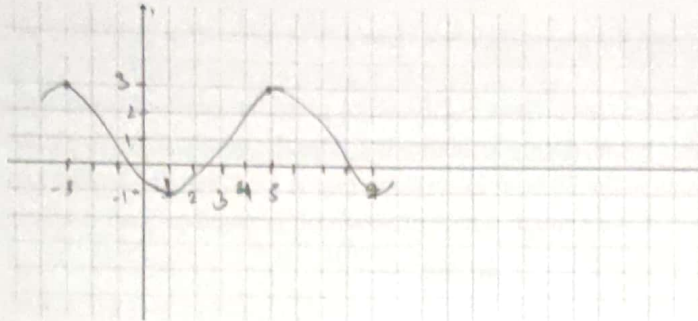
$$y = -2 \sin \left[\frac{2\pi(x-1)}{8} \right] + 1$$

3. Graph at least one cycle of each function.

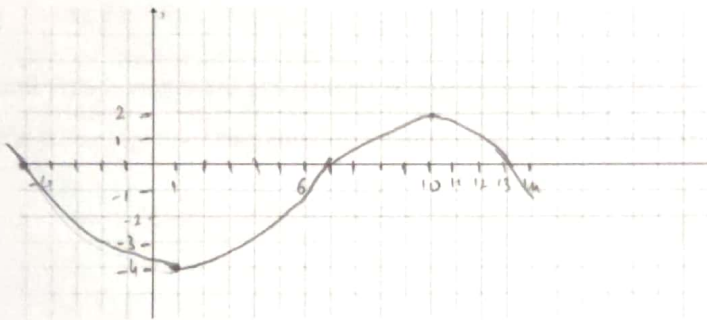
a) $y = \sin \frac{2\pi(x-5)}{12} + 1$



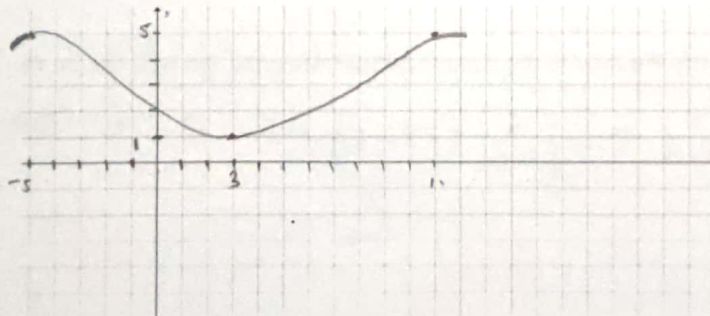
b) $y = 2 \cos \frac{2\pi(x+3)}{8} + 1$



c) $y = -3 \sin \frac{\pi(x+4)}{10} - 1$

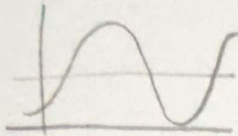


d) $y = -2 \cos \frac{2\pi(x-3)}{16} + 3$



Lesson EightApplications of Sinusoidal Functions

1. The function $h(t) = 4.3 \sin 0.15\pi(t - 2.5) + 6.4$ gives the depth of water at a seaport, h meters, at any time, t hours, during a certain day. Use the graph of the function to find the total number of hours in the 24 hour interval starting at $t = 0$ that the depth of the water is at least 6 meters deep. (Give a rough sketch of what you graphed on your graphing calculator).



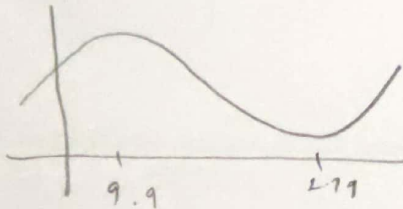
$$h \geq 6$$

$$\Rightarrow 4.3 \sin(0.15\pi(t - 2.5)) + 6.4 \geq 6$$

$$t = 14.12 \text{ hours}$$

2. In an Australian city the time of sunrise can be found using the trigonometric function $T = 4.6 \sin\left(\frac{\pi}{182}(d - 6)\right) + 5$ where T is the number of hours past midnight and d is the number of days into the year.

a) Sketch this function.



- b) Find algebraically the time to the nearest minute when the sun will rise on the 27th day of the year

$$4.6 \sin\left(\frac{\pi}{182}(27 - 6)\right) + 5 \Rightarrow 6:38 \text{ am.}$$

- c) Find how many days in the year that the sunrise is after 8 am.

$$8 = 4.6 \sin\left(\frac{\pi}{182}(d - 6)\right) + 5 \Rightarrow 100 \text{ days}$$

3. Ferris wheel has a diameter of 45 meters and its center is 43 meters off the ground. The wheel revolves once every 4 minutes. Lenny gets on the Ferris wheel at its lowest point at time zero.

a) Write a sinusoidal equation that would model Lenny's height above the ground as a function of time.

$$h(t) = -22.5 \cos\left(\frac{2\pi t}{4}\right) + 43$$

b) Determine when (after how many minutes) Lenny would first be 58 meters above the ground (solve algebraically or with calculator).

$$58 = -22.5 \cos\left(\frac{2\pi t}{4}\right) + 43$$

$$\Rightarrow t = 1.46 \text{ min}$$

4. A mass is supported by a spring so that it rests 25 cm above a table top. The mass is pulled down to a height of 10 cm above the table top and released at time $t = 0$. It takes 0.8 seconds for the mass to reach a maximum height of 40 cm above the table top. As the mass moves up and down, its height h , above the table top, is approximated by a sinusoidal function of the elapsed time t , for a short period of time. Determine a sine equation that gives the height, h , as a function of time, t .

$$h(t) = 15 \sin\left(\frac{2\pi(t-0.4)}{1.6}\right) + 25$$

Lesson Nine

Graphing the Tangent Function:

1. For each of the following state the period and the general equation for the asymptotes.

a) $y = \tan\left(x - \frac{\pi}{6}\right)$

Period = π

asymptote

$x = \frac{2\pi}{3} + k\pi$

b) $y = \tan 3x + 2$

Period = $\frac{\pi}{3}$

asymptote = $\frac{\pi}{6} + \frac{k\pi}{3}$

c) $y = \tan 2\left(x - \frac{\pi}{3}\right)$

Period = $\frac{\pi}{2}$

asymptote = $\frac{7\pi}{12} + \frac{k\pi}{2}$

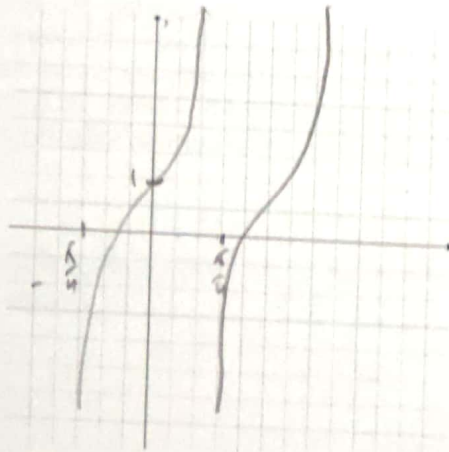
d) $y = \tan \frac{\pi}{4}(x + 1) - 1$

Period = 4

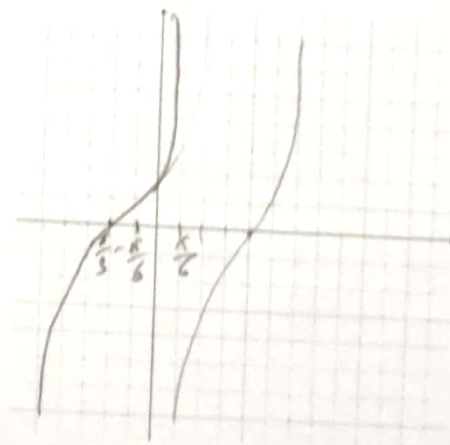
asymptote = $1 + 4k$

2. Graph at least two periods of each of the following functions. Clearly label the asymptotes.

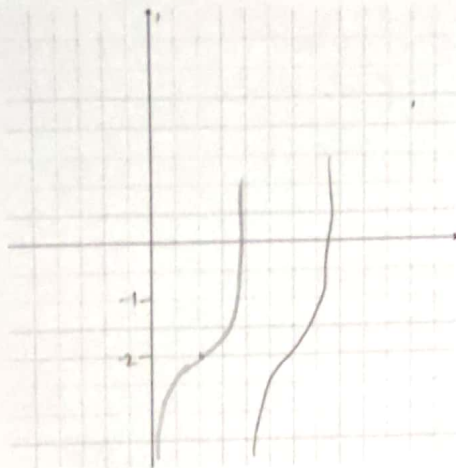
a) $y = \tan 2x + 1$



b) $y = \tan\left(x + \frac{\pi}{3}\right)$



c) $y = \tan 3\left(x - \frac{\pi}{6}\right) - 2$



d) $y = \tan \frac{\pi}{6}(x + 1)$

